ONE-TO-ONE LEARNING CENTER
the center of attention™

Page 1

B/A
A/B
B/A
A/B

Fill in what you know until you connect

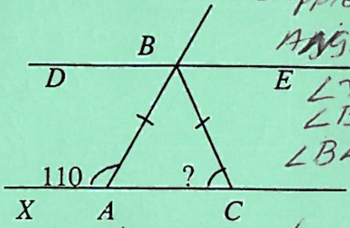
GEOMETRY

Parallel Lines are irrelevant

1. In the figure below, $\overline{AC}$ is parallel to $\overline{DE}$ with X on $\overline{AC}$ and B on $\overline{DE}$. Also $\overline{AB} \cong \overline{BC}$, and the measure of $\angle XAB$ is 110°. What is the measure of $\angle ACB$?

ISO

F. 35°
G. 40°
H. 55°
→ J. 70°
K. 110°



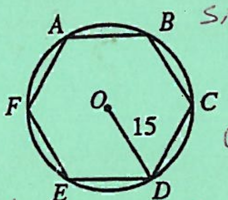
supplemental Angles
$\angle XAB + \angle BAC = 180°$
$\angle BAC = 70$

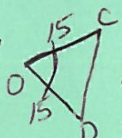
$\angle BAC = \angle BCA$ because base $\angle$'s =

2. Regular hexagon $ABCDEF$ is inscribed in a circle, as shown below. If the length of radius OD is 15 centimeters, how long is AB, in centimeters?

SINCE HEX is Regular, ALL sides are =

→ A. 15
B. 18
C. 30
D. 5π
E. $\frac{225\pi}{6}$





ΔOCD is equilateral so $CD = 15 = AB$

Angle $COD = 60°$ because there are ~~six~~ six equal opposite sides. $360 \div 6 = 60°$

3. A rectangle is twice as long as it is wide. If the width of the rectangle is 3 inches, what is the rectangle's area, in square inches?

TRANSLATE "Twice long as wide" means Length is Twice width.
$L = 2W$

A. 6
B. 9
C. 12
D. 15
→ E. 18

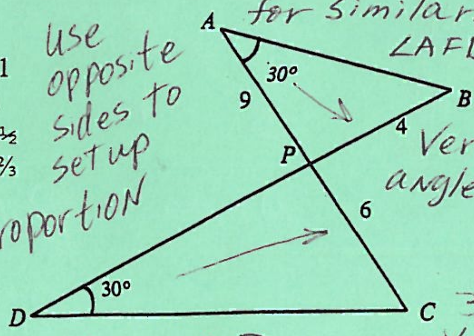
$\boxed{2W = 6}$ $W = 3$
$6 \times 3 = 18$

4. In the figure below, $\overline{DB}$ intersects $\overline{AC}$ at point P. The length of $\overline{AP}$ = 9 units, the length of $\overline{BP}$ = 4 units, and the length of $\overline{PC}$ = 6 units. How many units long is $\overline{DP}$?

NEED TWO EQUAL Angles for similarity.

A. 6
B. 911
C. 12
→ D. $13\frac{1}{2}$
E. $15\frac{2}{3}$

use opposite sides to set up proportion

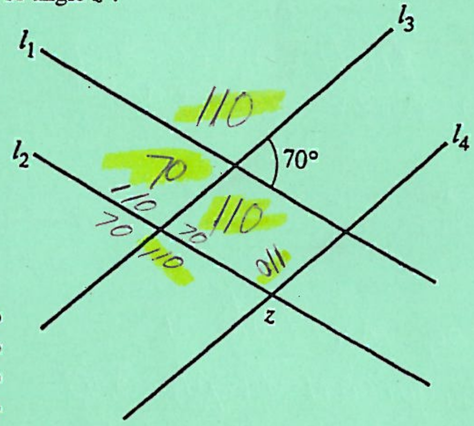


$\angle AFB = \angle DFC$ because Vertical angles are =

$$\frac{6}{4} = \frac{DP}{9} \quad DP = \frac{\overset{3}{\cancel{6}} \times 9}{\underset{2}{\cancel{4}}} = \frac{27}{2}$$

5. In the figure below, l^1 is parallel to l^2, l^3 is parallel to l^4, and the lines intersect as shown. What is the measure of angle z?

Fill in Pieces of the Puzzle

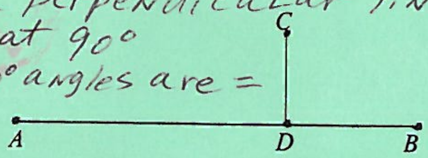


→ A. 110°
B. 120°
C. 130°
D. 140°
E. Cannot be determined from the given informa tion

usually not the correct choice

6. In the figure below, D is a point on $\overline{AB}$, and $\overline{CD}$ is perpendicular to $\overline{AB}$. Based on this information, which of the following is the best conclusion?

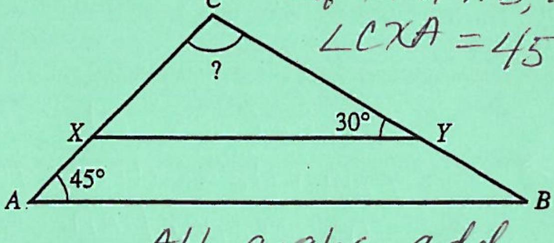
ALL perpendicular lines meet at 90°
ALL 90° angles are =



→ A. $\angle CDA \cong \angle CDB$.
B. $\angle CDA$ is larger than $\angle CDB$.
C. $\overline{AB}$ bisects $\overline{CD}$.
D. $\overline{CD}$ and $\overline{DB}$ are equal in length.
E. Point C is equidistant from A and B.

7. In the figure below, X and Y lie on the sides of ΔABC and $\overline{XY}$ is parallel to $\overline{AB}$. What is the measure of $\angle ACB$.

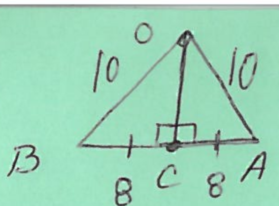
If $XY \parallel AB$, then $\angle CXA = 45°$



→ A. 105°
B. 115°
C. 125°
D. 135°
E. 150°

ALL angles add up to 180° in every triangle

$? = 180 - (45 + 30)°$
$= 180 - 75$
$= 105°$

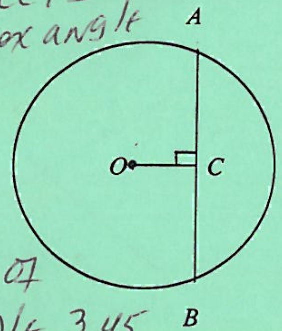


ALL Radii equaL so ΔBOA is isosceles

If there's No diagram, DRAW ONE

8. The circle shown below has a radius of 10 meters, and the length of chord $\overline{AB}$ is 16 meters. If O marks the center of the circle, what is the length of $\overline{OC}$?

ALTitude bisects BASE and VerTex angle



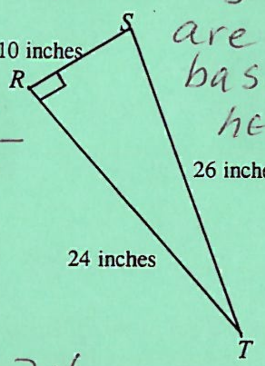
A. $2\sqrt{3}$
→ B. 6
C. 12
D. $4\sqrt{21}$
E. 36

Using multiple of Pythagorean triple 3,4,5
$\overline{OC} = 6$ because 6, 8, 10

9. In the triangle below, $\angle R$ is a right angle and the lengths of the sides are as marked. In square inches, what is the area of ΔRST?

The legs of a right Tri-angle are the base and height

$A = \frac{\text{base} * \text{height}}{2}$



F. 60
→ G. 120
H. 130
J. 240
K. 312

$\frac{10 * 24}{2} = 10 * 12 = 120$

10. The lengths of the sides of a triangle are 3, 8, and 9 inches. How many inches long is the shortest side of a similar triangle that has a perimeter of 60 inches?

→ F. 9
G. 11
H. 20
J. 24
K. 27

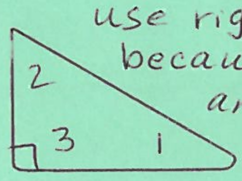
Perimeters of Similar triangles are equal in proportion

$\frac{\text{shortest side}}{\text{Perimeter}} = \frac{3}{(3+8+9)} = \frac{?}{60}$

$? = \frac{60 * 3}{20} = 3 * 3 = 9$

11. The ratio of the measures of the 2 smallest angles of a triangle is 1:2. and the ratio of the smallest to the largest angle is 1:3. What is the measure of the largest angle?

use right triangles because the 90° angle is always the largest.

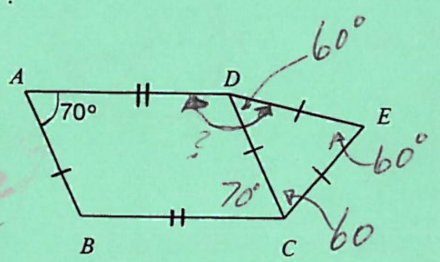


F. 45°
G. 77°
→ H. 90°
J. 108°
K. 135°

$1 + 2 + 3 = 6$ parts
$180° \div 6 = 30°$ The largest angle has 3 parts. so, $3 * 30° = 90°$

12. The figure below is made from a parallelogram, $ABCD$, and an equilateral triangle, ΔCDE. What is the measure of $\angle ADE$?

Opposite Angles in Parallegram are equal
ALL ∠'s in ▱ add up to 360°



A. 110°
B. 130°
C. 150°
→ D. 170°
E. 190°

$\angle ADC = 110° = \frac{360 - (70 + 70)}{2}$

$110° + 60° = 170°$

Diameter = 2 * radius
AREA of CIRCLE = πr^2

13. The diameter of a circle is 10 units long. What is the area of the circle, in square units?

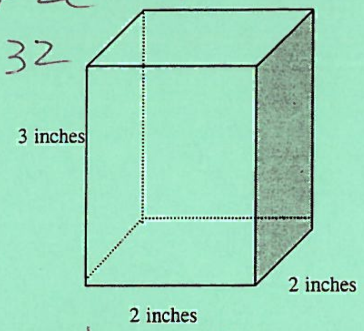
F. 5π
G. 10π
→ H. 25π
J. 100π
K. 400π

$10 = 2r$
$5 = r$
$\text{Area} = \pi(5)^2$
$= 25\pi$

14. The total surface area of the rectangular box shown below is the sum of the areas of the 6 sides. What is the box's total surface area, in square inches?

2(3*2) + 2(3*2) + 2(2*2) =

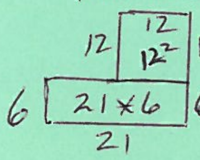
12+12+8 = 32



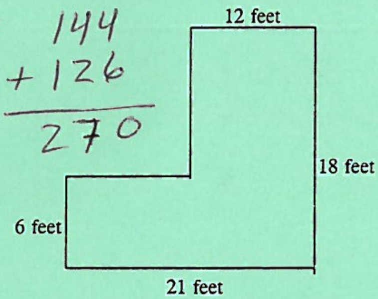
A. 12
B. 16
C. 20
D. 24
→ E. 32

(FRONT and BACK) + 2 sides + TOP and BOTTOM = TOTAL

15. A floor has the dimensions shown below. How many square feet of carpeting are needed to cover the entire floor?
(Note: All angles are right angles.)



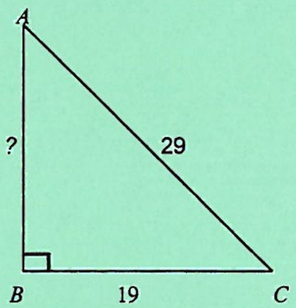
144
+126
270



A. 57
B. 72
C. 162
→ D. 270
E. 324

16. In the right triangle below, how long is side $\overline{AB}$?

Basic Pythagorean theorem

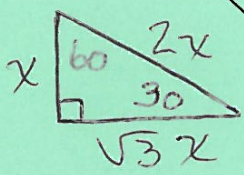


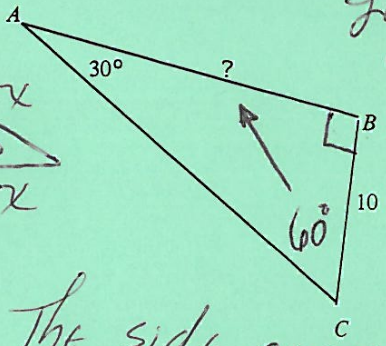
→ F. $\sqrt{29^2 - 19^2}$
G. $\sqrt{29^2 + 19^2}$
H. $29^2 - 19^2$
J. $29^2 + 19^2$
K. $29 - 19$

$(?)^2 = 29^2 - 19^2$

$? = \sqrt{29^2 - 19^2}$

17. In the figure below, $\angle B$ is a right angle and the measure of $\angle A$ is 30°. If $\overline{BC}$ is 10 units long, then how many units long is $\overline{AB}$?



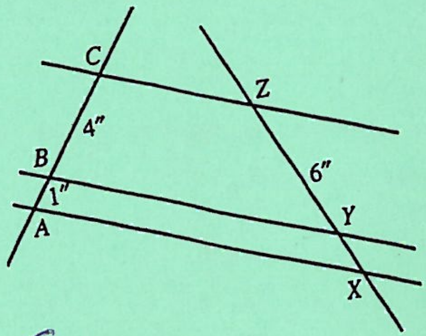


Label 90°

A. 5
B. 10
C. 20
D. $\sqrt{3}$
→ E. $10\sqrt{3}$

The side opposite 60° is $\sqrt{3}$ times larger than side opposite the 30° so $? = 10\sqrt{3}$

18. In the figure below, 3 parallel lines are crossed by 2 transversals as shown. The points of intersection and some distances, in inches, are labeled. What is the length, in inches, of YX?



F. 1
G. 2
H. 3
→ J. $\frac{3}{2}$
K. $\frac{5}{2}$

Segments cut by transversals of parallel Lines are equal in Proportion

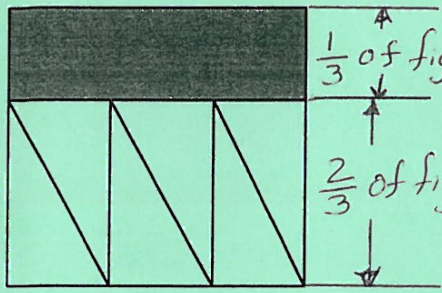
$\frac{4}{1} = \frac{6}{YX}$ so $YX = \frac{6*1}{4} = \frac{3}{2}$

Whole Area − Unshaded Area = Shaded Area

GEOMETRY II

1. The square below consists of a rectangle and 6 congruent triangles.

If $\frac{1}{3}$ is shaded then $\frac{2}{3}$ is not shaded



$\frac{1}{3}$ of figure

$\frac{2}{3}$ of figure

If the area of the shaded rectangle is ⅓ the area of the square, what fractional part of the total area of the square does the area of 1 of these triangles represent?

There are 6 triangles So one of them is $\frac{1}{6}$

F. $\frac{1}{9}$

G. $\frac{1}{6}$

H. $\frac{1}{3}$

J. $\frac{1}{2}$

K. $\frac{2}{3}$

$\frac{1}{6}$ of $\frac{2}{3} = \frac{1}{6} \times \frac{2}{3} = \frac{2}{18} = \frac{1}{9}$

sides and perimeters are equal in proportion in similar triangles

2. Triangle ΔABC is similar to ΔDEF. $\overline{AB}$ is 8 inches long, $\overline{BC}$ is 10 inches long, and $\overline{AC}$ is 16 inches long. If the longest side of ΔDEF is 40 inches long, what is the perimeter, in inches, of ΔDEF?

$\frac{\text{one side}}{\text{Perimeter}} = \frac{\text{Corresponding side}}{\text{Perimeter of other }\Delta}$

A. 74

→ B. 85

C. 90

D. 136

E. 170

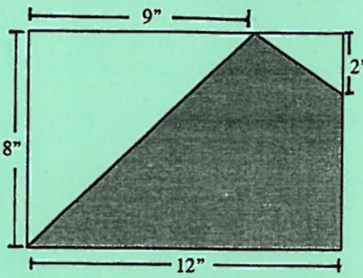
$\frac{\text{Longest side } \Delta ABC}{\text{Perimeter of } \Delta ABC} = \frac{\text{Longest side of } \Delta DEF}{\text{Perimeter of } \Delta DEF}$

$\frac{16}{34} = \frac{40}{?}$ so $? = \frac{\cancel{34}^{17} \times \cancel{40}^{5}}{\cancel{16}_{\cancel{2}\,1}}$

$? = 17 \times 5 = 85$

3. Lengths are shown in inches on the drawing of the rectangle below. What is the shaded area, in square inches?

Whole Area = 96
Unshaded −39
shaded 57



F. 18

G. 24

→ H. 57

J. 78

K. 96

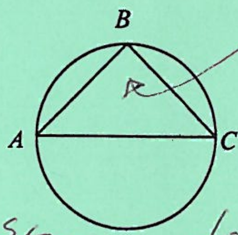
Large unshaded $\Delta = \frac{9 \times 8}{2} = 36$

Small unshaded $\Delta = \frac{3 \times 2}{2} = 3$

$36 + 3 = 39$

4. In the figure below, $\overline{AC}$ is a diameter of the circle, B is a point on the circle, and $\overline{AB} \cong \overline{BC}$. What is the degree measure of $\angle ABC$?

$\overset{\frown}{ABC}$ is a semicircle all angles inscribed a semicircle are 90°



always 90° if $\overline{AC}$ is diameter

A. 45°

B. 60°

C. 75°

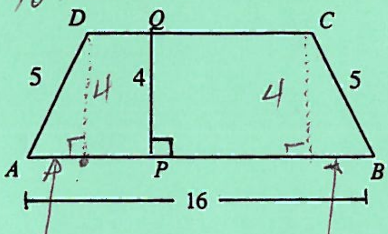
→ D. 90°

E. Cannot be determined from the given information

This choice is rarely the correct one.

5. In isosceles trapezoid $ABCD$ shown below, $\overline{QP}$ is an altitude, and all lengths are given in centimeters. What is the perimeter of trapezoid $ABCD$, in centimeters?

Move $\overline{QP}$ to form dotted lines to find PC



F. 30

G. 34

→ H. 36

J. 42

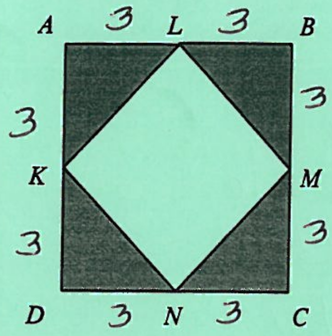
K. 52

$PC = 16 - (3+3) = 10$

Perimeter $= 10 + 5 + 5 + 16 = 36$

Mid point is half distance

6. Sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ of square $ABCD$ have midpoints L, M, N, and K, as shown below. If $\overline{AB}$ is 6 inches long, what is the area, in square inches, of the shaded region?

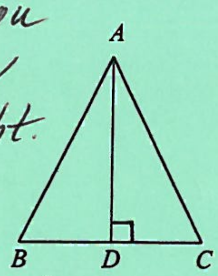


F. $4\frac{1}{2}$
G. $6\sqrt{2}$
H. 9
J. $12\sqrt{2}$
→ K. 18

$\left(\frac{3*3}{2}\right) * 4$ shaded Δ's $= 18$

7. In the figure below, $\overline{AB} \cong \overline{AC}$ and $\overline{BC}$ is 10 units long. What is the area, in square inches, of ΔABC?

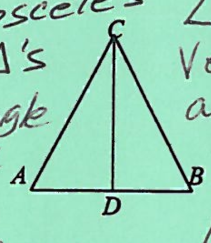
For Area, you need to know base and height. You only have base. Need to know height



F. 12.5
G. 25
H. $25\sqrt{2}$
J. 50
→ K. Cannot be determined from the given information

8. In the figure below, $\overline{CD}$ is an altitude of equilateral triangle ΔABC. If $\overline{CD}$ is $6\sqrt{3}$ units long, how many units long is $\overline{AC}$?

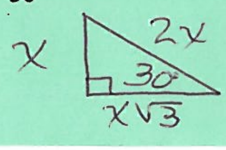
Altitude of isosceles and equilateral Δ's break vertex angle and base in half



$\angle ACB$ is Vertex angle and equals 60° because $180^\circ \div 3 = 60^\circ$

F. $3\sqrt{3}$
G. 6
→ H. 12
J. $12\sqrt{3}$
K. 36

Therefore ΔACD is a $30°/60°/90°$

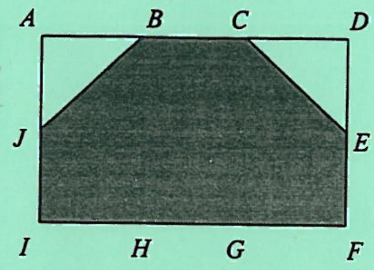


$CD =$ side opposite $60° = x\sqrt{3} = 6\sqrt{3}$
Therefore $x = 6$

$\overline{AC}$ is hypotenuse so its $2x$ or 12

9. In rectangle $ADFI$ below, the 10 labeled points are equally spaced along the perimeter. What is the ratio of the shaded area to the area of the entire rectangle?

If the points are equally spaced, the width is 2 and Length is 3

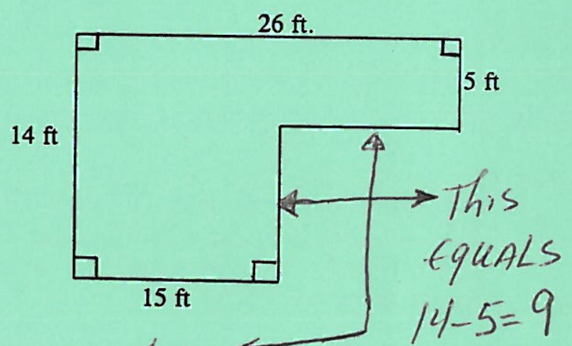


A. $\frac{7}{8}$
→ B. $\frac{5}{6}$
C. $\frac{4}{5}$
D. $\frac{3}{4}$
E. $\frac{2}{3}$

area of Rectangle $= 2*3 = 6$
shaded Region equals
$6 - 2\left(\frac{bh}{2}\right) =$
$6 - 2\left(\frac{1*1}{2}\right) = 6 - 1 = 5$

$\frac{\text{Shaded}}{\text{ENTIRE}} = \frac{5}{6}$

10. What is the perimeter, in feet (ft), of the figure below?

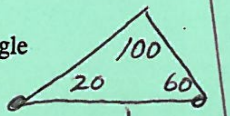


A. 60
B. 75
→ C. 80
D. 130
E. 364

$\text{Total} = 15 + 14 + 26 + 5 + 11 + 9 =$
80

11. If 2 interior angles of a triangle measure 20° and 60°, respectively, which of the following describes the location of the longest side of the triangle?

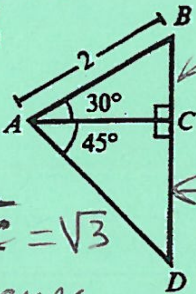
→ F. Always between the 20° and the 60° angle
G. Always opposite the 20°
H. Always opposite the 60° angle
J. Opposite either the 20° or the 60° angle
K. Cannot be determined from the given information



Longest side of triangle is always opposite largest angle

12. In the figure below, what is the length of *BD*?

The hypotenuse is Twice the length of the side opposite the 30° Angle



$\overline{BC}=1$ because its $1/2\ \overline{AB}$

$\leftarrow \sqrt{3}$

F. $\frac{1}{2}+\frac{\sqrt{3}}{2}$
G. $\frac{1}{2}+\sqrt{3}$
H. $1+\frac{\sqrt{3}}{2}$
→ J. $1+\sqrt{3}$
K. 2

$\overline{AC}=\sqrt{3}$ because $\overline{BC}=1$

If $\overline{AC}$ is $\sqrt{3}$, then $\overline{CD}=\sqrt{3}$ because 45° right triangles create isosceles Δ's. $1+\sqrt{3}=\overline{BD}$

13. The distance around a circular path is 1,000 meters. Which of the following most nearly approximates the radius of the path, in meters? (Note: $\pi \approx 3.14$)

A. 10
B. 18
C. 32
→ D. 159
E. 318

$C = D\pi = 2\pi r$
$1000 = 2(\pi)r$
$1000 = 2(3.14)r$
$\frac{1000}{6.18} = r = 161.81$
≈ 159

14. Each of the following determines a unique plane in 3-dimensional Euclidean space EXCEPT:

F. 1 line and 1 point NOT on the line.
G. 2 distinct parallel lines.
H. 2 intersecting perpendicular lines.
→ J. 2 lines intersecting in more than 1 point.
K, 3 distinct points NOT on the same line.

you need curved Lines for this To happen

15. Jamal will use a circle graph to show how he spends his time during a 24-hour day. The size of the sector representing each activity is proportional to the time spent in that activity. Among other activities, Jamal sleeps 9 hours. How many degrees should the central angle measure in the sector representing sleep?

→ A. 135°
B. 67½°
C. 37½°
D. 15°
E. 9°

$\frac{9 \text{ sleep hours}}{24 \text{ hours in day}} = \frac{3}{8}$

$\frac{3}{8}$ of $360° = 135°$

16. The volume of a sphere is given by the formula $V = 4/3\ \pi r^3$ and its surface area by the formula $S = 4\ \pi r^2$, where *r* is the radius of the sphere. What is the volume of a sphere, in cubic centimeters, if its surface area is 144π square centimeters?

A. 24π
B. 32π
C. 72π
D. 256π
→ E. 288π

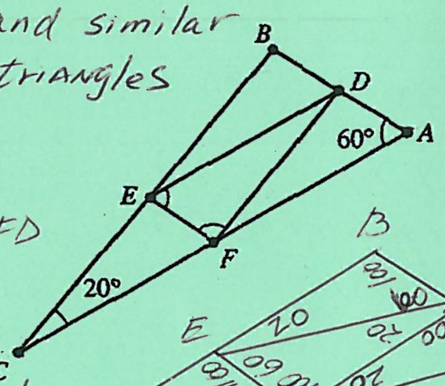
Discover radius by using surface area formula.

$144\pi = 4\pi r^2$
$6 = r$
$V = \frac{4}{3}\pi(6)^3 = 288\pi$

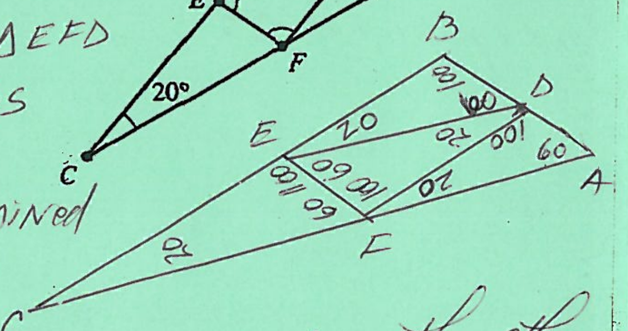
17. In the figure below, *D, E,* and *F* are the midpoints of the sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively. If the measure of $\angle BCA$ IS 20°, and the measure of $\angle BAC$ is 60. What is the sum of the measures of $\angle DFE$ and $\angle FED$?

Mid points when connected create parallel lines to sides and similar triangles

A. 60°
B. 80°
C. 100°
D. 120°
→ E. 160°



The ΔEFD Angles are determined by



supplementary angles, the other ones are determined by Similar Triangles

Here's a challenging cylinder problem that involves not only a unit conversion, but also the concept of proportion:

Q One hose dispenses water at the rate of one gallon per minute, and a second hose dispenses water at the rate of $1\frac{1}{2}$ gallons per minute. At the same time, the two hoses begin filling a cylindrical tank which is 14 inches in diameter and has a height of 10 inches. Which of the following most closely approximates the water level, measured in inches up from the tank's circular base, after $1\frac{1}{2}$ minutes? [231 cubic inches = 1 gallon]

(A) 3.5
(B) 4.2
(C) 4.8
(D) 5.6
(E) 6.7

A **The correct answer is (D).** After $1\frac{1}{2}$ minutes, the two hoses have dispensed a total of 3.75 gallons. Set up a proportion in which 3.75, as a portion of the tank's volume equals the water level after $1\frac{1}{2}$ minutes, as a portion of the tank's height:

$$\frac{3.75}{V} = \frac{x}{10}$$

The *volume* of the cylindrical pail is equal to the area of its circular base multiplied by its height:

$$V = \pi r^2 h = \frac{22}{7}(49)(10) = 1{,}540 \text{ cubic inches}$$

The *gallon* capacity of the pail = 1,540 ÷ 231, or about 6.7. Plug this value into the proportion. Then, solve for x:

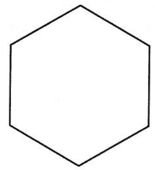
$$\frac{3.75}{6.7} = \frac{x}{10}$$
$$6.7x = 37.5$$
$$x = 5.6$$

BRAIN TEASER

In this quiz, you'll attempt ten tough ACT-style questions covering the topics from this lesson. Focus on applying the concepts and techniques you learned in this lesson, *not* on answering the questions as quickly as possible. Then, read the explanations that follow the quiz, even for the questions you answered correctly.

QUIZ

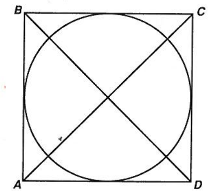
1.



If each side of the hexagon shown above is one meter in length, what is the area of the hexagon?

(A) $\frac{2\sqrt{3}}{3}$
(B) $\sqrt{3}$
(C) $\frac{3\sqrt{3}}{2}$
(D) $2\sqrt{2} + 1$
(E) 4

2.



If the circumference of the circle shown above is 16π, and if AC equals BD in length, what is the length of AC?

(F) 12
(G) $8\sqrt{2}$
(H) 16
(J) $12\sqrt{3}$
(K) $16\sqrt{2}$

3. If all three vertices of a triangle lie along a circle's circumference, and if one of the triangle's sides is equal in length to the circle's diameter, what is the largest possible perimeter of the triangle, in terms of the circle's diameter (d)?

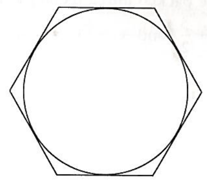
(A) $\frac{4}{3}d$
(B) $\frac{\pi d}{2}$
(C) $d\sqrt{2} + d$
(D) $d + \frac{2\sqrt{3}}{d}$
(E) $d(d + \sqrt{3})$

4. If the diameter of a circle increases by 50%, which of the following statements is true?

I. The circle's circumference increases by 50%.
II. The circle's radius increases by 100%.
III. The circle's area increases by 150%.

(F) I only
(G) II only
(H) I and II only
(J) I and III only
(K) I, II, and III

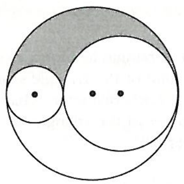
5.



A circle is circumscribed by a hexagon whose sides are all equal in length, as shown above. If the area of the circle is 3π, what is the area of the hexagon?

(A) $2\pi\sqrt{2}$
(B) $6\sqrt{3}$
(C) $\frac{11}{3}\pi$
(D) $9\sqrt{2}$
(E) $\frac{9}{2}\pi$

6.



In the figure above, the centers of all three circles lie on the same line. The radius of the middle-sized circle is twice that of the smallest circle. If the radius of the smallest circle is 1, what is the length of the boundary of the shaded region?

(F) 9
(G) 3π
(H) 12
(J) 6π
(K) 12π

7. If a circle with a radius of x has an area of 4, what is the area of a circle with a radius of $3x$?

(A) 40
(B) 36
(C) 28
(D) 24
(E) 20

8. If s is an integer greater than 1, how many entire 1-inch cubes can be packed into a rectangular box having sides s, $s+\frac{3}{2}$, and $s-1$, measured in inches?

(F) $s^3 - s$
(G) $s^3 + \frac{s^2}{2} + \frac{s}{2}$
(H) $s^3 - 2s + s$
(J) $s^3 + s^2 - s$
(K) None of the above

9. The volume of a cube, each face of which has an area of 16 square inches, equals the volume of a right cylinder with a height of 16 inches and a circular base. Which of the following most closely approximates the diameter of the cylinder's base?

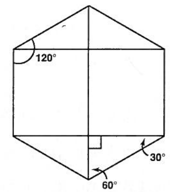
(A) $1\frac{1}{5}$ inches
(B) $2\frac{1}{4}$ inches
(C) 4 inches
(D) $6\frac{1}{2}$ inches
(E) 8 inches

10. The volumes of two rectangular solids having the same proportions are 250 and 128. If one edge of the larger solid is 25 centimeters in length, what is the centimeter length of the corresponding edge of the smaller solid?

(F) 34
(G) 30
(H) 27.5
(J) 20
(K) 14.4

ANSWERS AND EXPLANATIONS

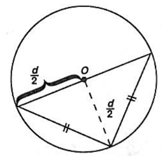
1. **The correct answer is (C).** The sum of the measures of the six interior angles must total 720°. Since the hexagon's sides are all the same length, the six angles are all equal in mesure: 120°. You can divide up the figure as indicated here:



Each of the four triangles is a 30°-60°-90° triangle, so the ratio of the sides of each is $1:\sqrt{3}:2$. The hypotenuse is 1, so the other two sides are $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$. The area of each triangle $= \frac{1}{2} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$. The hexagon includes four such triangles, so their total area is $\frac{\sqrt{2}}{2}$. The area of each of the two rectangles is $1 \times \frac{\sqrt{3}}{2}$, so the area of both rectangles combined is $\sqrt{3}$. The total area of all triangles and both rectangles is $\sqrt{3} + \frac{\sqrt{3}}{2}$, or $\frac{3\sqrt{3}}{2}$.

2. **The correct answer is (K).** First find the circle's diameter. Given the circle's circumference of 16π, its diameter = 16. Thus, each of the square's sides = 16. The ratio of the square's side to its diagonal is $1:\sqrt{2}$. (Remember the Pythagorean triplet $1:1:\sqrt{2}$ from the previous lesson?) Thus, diagonal $AC = 16\sqrt{2}$.

3. **The correct answer is (C).** The largest possible triangle meeting the stated criteria is an isosceles triangle, in which the triangle's third point lies midway between the other two along the circle's circumference. You can determine this by visualization and a bit of logic. As you move the third vertex away from that midway point, the triangle's perimeter and area decrease (the perimeter approaching the circle's diameter while the area approaches zero). To determine the length of each of these triangle's legs, divide the triangle into two right triangles, each conforming to the Pythagorean triplet $1:1:\sqrt{2}$ (the two legs each equal the circle's radius, or $\frac{d}{2}$):



Applying the same triplet to the large triangle, in terms of d each leg of the *large* triangle $= \frac{d\sqrt{2}}{2}$. Thus, you can express the triangle's perimeter (the sum of all the sides) as:

$$\frac{d\sqrt{2}}{2} + \frac{d\sqrt{2}}{2} + d = \frac{2d\sqrt{2}}{2} + d = d\sqrt{2} + d$$

What about working backward from the answer choices by plugging in a simple value for *d*? This strategy would not help much, would it? You'd still need to analyze the problem as shown above. (Remember: Working backward helps for some problem solving questions but not for others.)

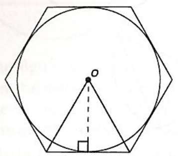
4. **The correct answer is (F).** If you forgot the ratios you learned in this lesson, try plugging in a simple number, such as 2, for d. Here's the analysis of each statement (let $d = 2$, or $r = 1$):

(I) is true. $C = \pi d = 2\pi$. The new circumference $= (1.5)(2\pi) = 3\pi$. The new circumference is $\frac{3}{2}$ the circumference of (or 50% greater than) the original one.

(II) is false. $r = \frac{d}{2} = 1$. The new diameter $= 2(1.5) = 3$. (The new diameter is 1.5 times the original one (or 50% greater than) the original one. The new radius is $\frac{d}{2}$=1.5, 1.5 times (or 50% greater than, not 100% greater than) the original one.

(III) is false. $A = \pi r^2 = \pi 1^2 = \pi$. The new area $= \pi\left(\frac{3r}{2}\right)^2 = \frac{9}{4}\pi$. The new area is $\frac{9}{4}$ the area of (or 125% greater than) the original one.

5. **The correct answer is (B).** Construct two right triangles as shown in the next figure. In each right triangle, the angles measure 30°, 60°, and 90° (one of the Pythagorean angle triplets). Accordingly, the triangle's sides conform to the Pythagorean side triplet $1:\sqrt{3}:2$.



Given that the circle's area is 3π, its radius (which is also the longer leg of the right triangle) equals $\sqrt{3}(3\pi = \pi r^2)$. Accordingly, the area of each right triangle $= \frac{1}{2}bh = \frac{1}{2}(1)\left(\sqrt{3}\right) = \frac{\sqrt{3}}{2}$. The hexagon is comprised of 12 of these right triangles, all having the same area. Thus, the hexagon's area $= \frac{12\sqrt{3}}{2} = 6\sqrt{3}$.

6. **The correct answer is (J).** Since the smallest circle has a radius of 1, the medium circle has a radius of 2, and, therefore, the diameter of the large circle must be 6, which makes its radius 3. The arc of a semi-circle is half the circle's circumference—that is, πr. So, the length of the boundary of the shaded region is the sum of the arcs of the three semi-circles: $\pi + 2\pi + 3\pi = 6\pi$.

7. **The correct answer is (B).** The area of a circle is πr^2. The area of a circle with a radius of x is πx^2, which is given as 4. The area of a circle with radius $3x$ is $\pi(3x)^2 = 9\pi x^2$. Therefore, the area of the larger circle is 9 times the area of the smaller circle.

8. **The correct answer is (F).** The edge $\left(s + \frac{3}{2}\right)$ only accommodates $(s + 1)$ 1-inch cubes along its edge. The additional half-inch is unused space. Thus, the number of 1-inch cubes that can be packed into the box is the product of the three edges: $(s)(s+1)(s-1) = s(s^2 - 1) = s^3 - s$.

9. **The correct answer is (B).** Since the area of each square face of the box is 16 inches, each edge is $\sqrt{16}$, or 4, inches in length. Accordingly, the volume of the box is 4^3, or 64, cubic inches. Apply the formula for a cylinder's volume ($V = \pi r^2 h$), solving for radius (r):

$$64 = (\pi r^2)(16)$$
$$4 = \pi r^2$$
$$\frac{4}{\pi} = r^2$$
$$\frac{2}{\sqrt{\pi}} = r$$

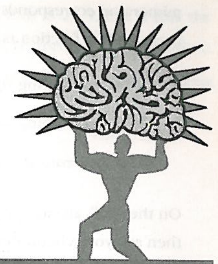
The diameter of the circular base is twice its radius, or $\frac{4}{\sqrt{\pi}}$. Using 1.8 as the approximate value of $\sqrt{\pi}$ yields a diameter of approximately $2\frac{1}{4}$ inches.

10. **The correct answer is (J).** Since the two solids are proportionately identical, the ratio of the volumes is equal to the cube of the linear ratio of each pair of corresponding edges. The ratio of the two volumes can be expressed and simplified in this way: $\frac{250}{128} = \frac{125}{64}$. From here, you can determine that the linear ratio of the two edges is 5 to 4:

$$\frac{\sqrt[3]{125}}{\sqrt[3]{64}} = \frac{5}{4} \text{ (or 5:4)}$$

Using the proportion method, set up an algebraic equation to solve for the length of the smaller edge (x):

$$\frac{5}{4} = \frac{25}{x}$$
$$5x = 100$$
$$x = 20$$



LESSON 14

Coordinate Geometry and Trigonometric Graphs

In this lesson, you'll learn how the test-makers design brain-taxing questions involving the standard (x,y) coordinate plane. Here are the specific topics you'll cover:

- Defining and plotting lines on the (x,y) plane
- Applying the midpoint and distance formulas to problems involving simple 2-dimensional figures
- Understanding the equations of conic sections (circles and ellipses) and of parabolas and hyperbolas—and their corresponding graphs on the (x,y) plane
- Graphing trigonometric functions on the (x,y) plane

All of these topics build on concepts covered in earlier lessons, making the material you'll encounter in the pages ahead more advanced.

Equations and graphs of ellipses, parabolas, and hyperbolas, as well as trigonometric graphs, are key areas that distinguish the ACT from the "whimpier" SAT, which doesn't cover any of these topics.

DEFINING A LINE ON THE COORDINATE PLANE

You can define any line on the coordinate plane by the equation $y = mx + b$. In this equation:

- m is the slope of the line
- b is the y-intercept (where the line crosses the y-axis)
- x and y are the coordinates of any point on the line
- Any (x,y) pair defining a point on the line can substitute for the variables x and y

Determining the *slope* of a line is usually crucial to solving ACT problems of this type. Think of the slope as a fraction whose numerator indicates the vertical change from one point to another on the line (moving left to right). The